Overdale Junior School

Written Calculation Policy

October 2014

1. Introduction

This policy shows the progression of teaching written calculations at Overdale Junior School. Each method is designed to lead on to the next, gradually building up children's understanding of number. The use of Numicon as a visual tool is crucial to support learning.

Written methods for addition of whole numbers
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for each operation which they know they can rely on when mental methods are not appropriate. These notes show the stages in building up to using an efficient written method by the end of Year 6.

Note: It is important that children's mental methods of calculation are practiced on a regular basis and secured alongside their learning and use of an efficient written method for addition.

Using and Applying
Before children move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts (including money, time and other measures).

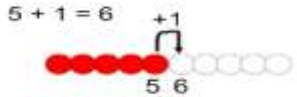
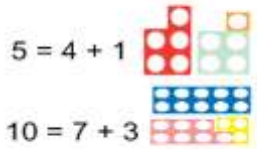
Please feel free to come and have a chat about calculation methods. I am available most Tuesday mornings.

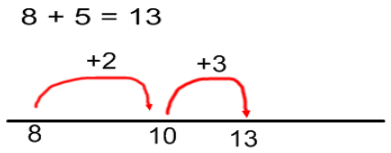
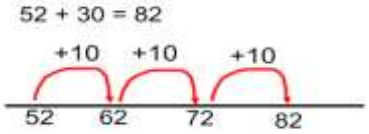
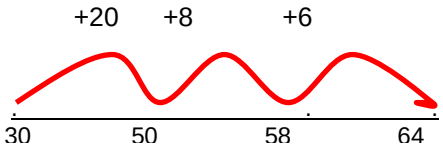
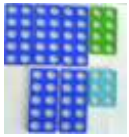
Miss Sultana
(Maths Co-ordinator)

Date of issue: October 2014

ADDITION
This shows the progression of teaching written calculations for addition.

These notes show the stages in building up to using an efficient written method for addition of whole numbers by the end of Year 4.

STAGE	SKILLS	METHODS – CONCEPT & IMAGES	COMMENTS
STAGE 1 Add and subtract one-digit and two-digit numbers to 20, including zero	Counting Using conservation of number.	Combining groups of objects to find the total 	Put all objects together and count… Find total of 2 groups using objects in hoops… Then total of 2 groups using objects and numerals in hoops… Then… total of 2 groups using objects and hoops and recording as a number sentence… Then without hoops, with objects and record as a number sentence
	Finding numbers Relating groups of objects to number line.	'Informal number line' / number sentences As above, alongside a calculation 	Look at number sentences. Use objects on sheets to find answer Then… Look at number sentences – use objects provided to find the answer Look at number sentences: what do we have to do? Use objects to find an answer
	Counting to ten and back Locating numbers on a number line & adding one more.	Add one onto a number What is the effect of adding 1? 	Find 5 on number track, then add one Encourage children to locate the first number and count on from there, rather than starting at zero. Pegs on a coat hanger (turn round to show inverse) Write all the ways to make 2,3,4,5,6,7,8,9,10
	- recall all addition pairs to 9 + 9 and complements in 10	How many ways of partitioning a number? 10 = ? + ? 9 = ? + ? 8 = ? + ? Etc 	Model with Numicon In order to calculate effectively children must know all the bonds for numbers up to ten. This will enable them to jump on the number line rather than count. Using Numicon is also an effective way to show how to split smaller numbers up.

STAGE	SKILLS	METHODS – CONCEPT & IMAGES	COMMENTS
STAGE 2 / 3 add and subtract numbers using concrete objects, pictorial representations, and mentally, including: -a two-digit number and ones -a two-digit number and tens -two two-digit numbers -adding three one-digit numbers	Rapid recall Adding to a ten mentally (10 + 2 = 12, 10 + 3 = 13 10 + 4 = 14…) Using number line to jump in tens from any 2-digit number. Adding on the number line.	Bridge 10 (e.g. 8 + 5 =13) 	Use concrete objects to relate addition calculation to written number calculation. Emphasise JUMP on number line, NOT counting! Use number bonds to jump to the next ten on the number line. Then add what is left in one jump. Using number bonds to add on the number line.
	Counting on and back in steps of ten.	 Adding multiples of 10 Adding 3 one digit no.s 	Starting from any 2-digit number children must be able to jump in steps of ten. Focus on what happens to the tens and units as you count. Focus on tricky parts: counting over 100, counting back past 20 in the teen numbers. Children become efficient at calculating to 100,1000, etc.
	Counting on in tens. Adding to a ten mentally. Adding tens first, then ones.	TU + TU 38 +26   30+20+8+6 = 64	This puts together the two previous ways of adding on a number line. THE NUMBER LINE REPRESENTS THE JUMPS IN YOUR HEAD! **Adding tens first**, then ones, will help progression to the next stage of partitioning. If adding near multiples of ten, more confident pupils can do adding a ten and adjusting: 43 + 19, = 43 + 20 = 63 -1 = 62

<table>
<tr><th>STAGE</th><th>SKILLS</th><th colspan="2">METHODS – CONCEPT & IMAGES</th><th>COMMENTS</th></tr>
<tr><td rowspan="2"><u>STAGE 3 / 4</u>
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction</td><td>Partition number into tens and units.
Place into place value columns.
Start with significant digit.</td><td colspan="2">425 400 +20 + 5
+ 86 80 + 6
400 (400 + 0)
100 (20 + 80)
11 (5 + 6)
511</td><td>Progression to formal written method
The mental calculation above will aid their understanding of this formal calculation.
Children estimate the solution.
Children partition the number into H T U.
They need to be secure with their PV knowledge.
They translate the partitioned numbers into columns.
Children decide appropriate method depending on size of number.</td></tr>
<tr><td>- Partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.</td><td>HTU + HTU using partitioning
347 + 122 =
300 40 7
+100 20 2
400 60 9 = 469</td><td>THEN, GO BEYOND 10 in U column etc.
159 + 264 =
100 50 9
+ 200 60 4
300 110 13 = 423</td><td>Start by partitioning the numbers so the children understand what each column represents.

Children should only use this when adding together 3-digit numbers and preferably when the units add to more than ten (although to introduce concept using simpler numbers is a good idea).</td></tr>
<tr><td rowspan="2"><u>STAGE 4 / 5</u>
Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.

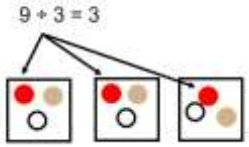
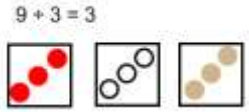
<u>STAGE 5 / 6</u>
Pupils practice addition, for larger numbers, using formal written methods of columnar addition.</td><td rowspan="2">Compact Column addition

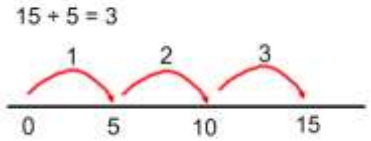
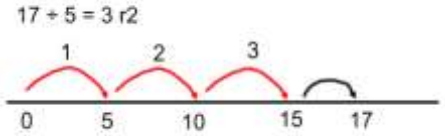
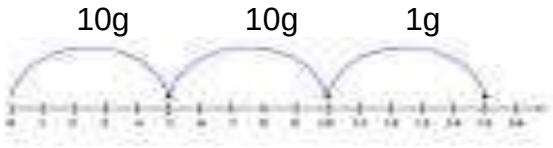
Compact Column addition with decimals</td><td>347 + 122
From 347
+122
469</td><td>Then, with carrying
159
+ 264
423
11</td><td rowspan="2">As the children become more confident in column addition they can gradually start to use the compact method for speed.
It is vital that they still understand that the small ‘1’ represents tens or hundreds.
Discussion around starting the addition process with the least significant digit is necessary. As with the compact column addition strategy it is vital that children understand what each column represents in terms of value.
‘Moving digits’ ITP to investigate decimals.
Practice partitioning.
Number bonds up to ten (to avoid counting in ones when adding up columns).</td></tr>
<tr><td>Same number of decimal places
78.5 km
+54.6 km
133.1 km
1 1</td><td>Then, different number of decimal places
124.9
+ 7.25
132.15
1 1</td></tr>
</table>

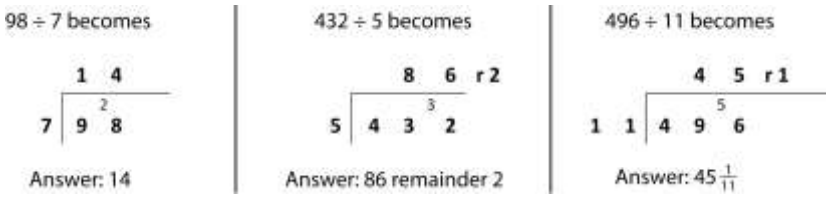
DIVISION

This shows the progression of teaching written calculations for division.

Each method is designed to lead on to the next gradually building up children's understanding of number. Remember, children should think about whether a calculation can be done mentally first.

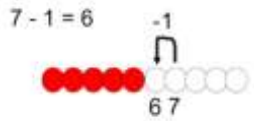
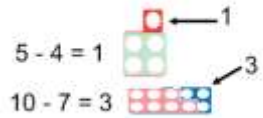
STAGE	SKILLS	METHODS – CONCEPT & IMAGES	COMMENTS
STAGE 1 Solve one-step problems involving division.	**Sharing** **Counting in groups.**	SHARING 'Is it fair?' 	USE COUNTERS OF DIFFERENT COLOURS When sharing you know how many groups you will have; you are working out how many will be in each group. **Don't 'over – teach' sharing! -** Focus more on grouping. Calculate the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
STAGE 2 Solve problems involving division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts.	**Grouping** Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.	As GROUPING – link to times tables facts 12 ÷ 4 = 3 (groups)  Repeated subtraction	When solving division through grouping, you know how many items are in each group; you are working out how many groups there will be. As this relies more on times **tables knowledge**, it is better to use this strategy than sharing. Children should understand that even when solving a 'sharing' problem, they can solve it quicker through **grouping**. Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs Solve problems in contexts.

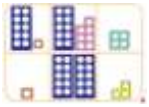
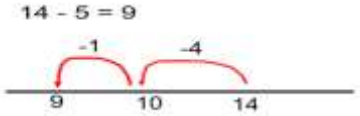
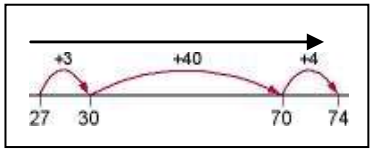
STAGE	SKILLS	METHODS – CONCEPT & IMAGES	COMMENTS
STAGE 3 Write and calculate mathematical statements for division including for two-digit numbers times one-digit numbers. Grouping on the number line with remainders.	**Grouping on the number line** Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.	**Grouping using number line** 15 ÷ 5 = 3  Finding a remainder 17 ÷ 5 = 3 r 2  	Encourage children to read the question as: 'I have 15, how many 5s?' They can then use times tables knowledge to solve the problem, using number lines to record their thinking. using mental and progressing to formal written methods. Encourage children to read the question as: 'I have 17, how many 5s?' How many WHOLE groups of 5 can they count in 17? What's left over? This is the remainder. Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and **progressing** to the formal written methods of short multiplication and division.
STAGE 4 / 5 Pupils **practise to become fluent** in the formal written method of short division with exact answers.	Grouping on number line with larger numbers. Can find other known facts 10x 20x 30x 2x5x Recall multiplication and division facts for multiplication tables up to 12 × 12 Continue to practise recalling and using multiplication tables and related division facts to aid fluency.	 240 24 504÷24 480 504 answer= 21 Use known multiplication facts alongside to help with jumps Eg; 10x24=240 5x24=120 2x24= 48	Write known facts alongside number line to help with jumps. Write totals underneath points on the number line to know where to jump to next. Pupils practise mental methods and extend this to three-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6). Pupils write statements about the equality of expressions (for example, use the distributive law 39 × 7 = 30 × 7 + 9 × 7 -and associative law (2 × 3) × 4 = 2 × (3 × 4)). They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, 2 x 6 x 5 = 10 x 6 = 60.

STAGE	SKILLS	METHODS – CONCEPT & IMAGES	COMMENTS
STAGE 5/ 6 Divide numbers up to 4 digits by a one-digit number using the formal written method of short division.	Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.	Formal 'Goes Into' Method 5 | 72 = 14 r 2 (2) Then 6 | 987 = 164 3/6 (3 2)	Using the standard 'goes into' method allows children to use known multiplication facts mentally and reduce the jottings needed to record their thoughts. Interpret remainders appropriately for the context. How many 5s go into 7? =1, carry 2 tens over to ones column. How many 5s go into 22? =4 r2
STAGE 6 Decimal Divisions Divide numbers up to 4 digits by a two-digit number using the formal written method of short division. Use written division methods in cases where the answer has up to two decimal places.	Knowledge of decimal place value. Times tables- rapid recall.	With decimals: Use formal method 87.5 ÷ 7 = 12.5 (1 3) 7 | 8 7 . 5 More able could express answer to calculation as **12.5**, or **remainder 3.5,** discussing that 3.5 left over is half of the 7 that they were dividing the number by.	Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding. (For example, 98 ÷ 4 = 98/2 = 24 r 2 = 24 1/2 = 24.5 ≈ 25). where appropriate, interpreting remainders according to the context Pupils are introduced to the division of decimal numbers by one-digit whole number, initially, in practical contexts involving measures and money.
			

SUBTRACTION
This shows the progression of teaching written calculations for subtraction.

These notes show the stages in building up to using an efficient written method for subtraction of numbers by the end of Year 5.

STAGE	SKILLS	METHODS – CONCEPT & IMAGES	COMMENTS
Early subtraction	Counting	Take away a number of objects from the group, count what's left 	Then… start with group of objects and record the numeral. Take some away, record and count what's left (record). '6 take away 3 is 3 OR 3 less than 6 is 3
	Finding numbers. Relating groups of objects to number lines. Compare two quantities.	Introduce – and = symbols Include vocabulary: 'difference' Relate to number line 	Emphasise JUMPING on number line, not counting. Then… look at a number line: what do we need to do?
	Counting to ten and back. Locating numbers on a number line and finding one less.	Subtract one from a number What is the effect of subtracting 1? 	Find 5 on number track, then SUBTRACT one. Encourage children to locate the first number and count back from there, rather than starting at zero.
	Know number bonds to ten.	Inverse use of number bonds  Reason 10-7=3 3+7=10	**Model with Numicon** In order to calculate effectively children must know all the bonds for numbers up to ten. This will enable them to **jump** back on the number line rather than *count*. Using a Numicon is an effective way to show how to split smaller numbers up. **KS1 children to also model this using jumps on a number line in order to lead to stage 1.** Use pegs on washing line (reverse to show inverse).

STAGE	SKILLS	METHODS – CONCEPT & IMAGES		COMMENTS
STAGE 1 represent and use number bonds and related subtraction facts within 20 - add and subtract one-digit and two-digit numbers to 20, including zero	Partition numbers using their place value. Use number bonds to jump back on the number line. - recall all addition and subtraction facts to 20.	Jumping back (Bridging 10)  Numicon tiles  Modelling 14-5= 9 9+5=14		Emphasise JUMP on number line, NOT counting! Use number bonds to jump back to the previous ten on the number line. Then subtract what is left in one jump. Model by overlaying Numicon tiles. Lay out a ten and four tile so they join together to make 14. Place a five tile over the top of the four tile and part of the ten tile to show a subtraction of five. Look to see how much of the blue ten tile is left.
STAGE 2 / 3 subtract numbers including: - a two-digit number and ones - a two-digit number and tens -two two-digit numbers - adding three one-digit numbers	Starting from any 2-digit number, jump on in steps of one and ten. If subtracting near multiples of ten, more confident pupils can do subtracting a ten and adjusting: 43 - 19, = 43 - 20 = 23 +1 = 24	Model this with cube towers.Looking at the difference between 2 towers.Relate no. of cubes to a no. line.  Subtract numbers using concrete objects, pictorial representations, and mentally.	Jumping on in tens.74 – 27  Don't use number line for HTU – HTU (only exception is something like 1,000 – 279, which would involve too many exchanges).	Emphasise subtraction as 'how much less' and 'finding the difference' to progress to the next step. Counting on is easier than counting back. Model on a number line how taking away the beginning of the number line for a subtraction ('how much less'), is the same as counting on from the smaller number to the larger number in the calculation (finding the difference). *Focus on tricky parts: counting over 100, counting back past 20 in the teen numbers*. **Emphasise looking at HOW CLOSE NUMBERS ARE before diving into use of a number line.** The children should question: Is it a good idea to take away? (Build towards two steps on no. line).
Children develop efficiency through mentally adding to the next multiple of 10/100/1000 etc.				
When children are accurate and efficient with this method of subtraction then alternative methods may be introduced, such as decomposition and negative number.				

<table>
<tr><th>STAGE</th><th>SKILLS</th><th colspan="2">METHODS – CONCEPT & IMAGES</th><th>COMMENTS</th></tr>
<tr><td rowspan="2"><u>STAGE 3</u>
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.

<u>STAGE 4</u>
up to 4 digits</td><td rowspan="2">Can use knowledge of extra partitioning
e.g. 65 is…
60 + 5 or
50 +15 or
40 + 25 or…
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value.</td><td>536-215

536
-<u>215</u>
+85 (->300)
+200 (->500)
<u>+ 36</u> (->536)
321</td><td>Children could investigate the same method of subtracting as stage 2 but transfer this from the number line to layout in column.

Ideally children should only be using column method when practicing decomposition.</td><td rowspan="2">Encourage efficiency in calculating the difference in two steps.
Starting with the expanded method is the best way to get children to understand what is happening when using column subtraction.
Get them to understand that if you can’t subtract the units exchange a ten, and so forth.
MISCONCEPTION: Children often try to swap the units if they can’t subtract them properly first so model this carefully. Model using Dienes apparatus.

- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).</td></tr>
<tr><td colspan="2">HTU – HTU Using decomposition

536 – 215

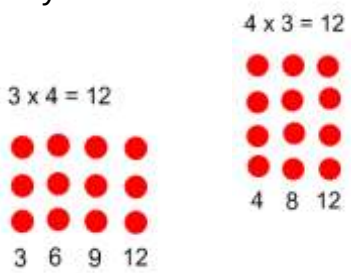
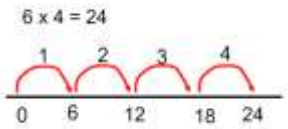
500 30 6
- <u>200 10 5</u>
300 20 1 = 321

Then 536 – 277=
400 120 1
- <u><s>500</s> <s>30</s> 6</u>
200 70 7
200 50 9 = 259</td></tr>
<tr><td><u>STAGE 5</u>
Compact column subtraction
More than 4 digits.</td><td>Use knowledge of place value when carrying.</td><td colspan="2">Compact column subtraction
2 1 1
1 <s>3</s> 7 Then… <s>5</s> <s>3</s> 6
- 2 9 - 2 7 7
1 0 8 2 5 9</td><td>As the children become more confident in column subtraction they can gradually start to use the compact method for speed.
It is vital that they still understand that the small ‘1’ represents tens or hundreds.</td></tr>
<tr><td><u>Stage 5 / 6</u>
Compact column subtraction with decimals.</td><td></td><td colspan="2">With decimals
1
6 1 1
<s>7</s> <s>2</s> . 5 km Then… 1 13 10 1
- 4 . 6 km 3 <s>2</s> <s>4</s> . <s>1</s> 2
6 7 . 9 km - 7 . 2 5
3 1 6 . 8 7</td><td>As with the compact column subtraction strategy it is vital that children understand what each column represents in terms of value.
‘Moving digits’ ITP to investigate decimals.</td></tr>
</table>

MULTIPLICATION

This shows the progression of teaching written calculations for multiplication.

Each method is designed to lead on to the next gradually building up children's understanding of number. Remember, children should think about whether a calculation can be done mentally first.

STAGE	SKILL	METHOD-CONCEPT & IMAGES		COMMENTS
STAGE 1 Solve one-step problems involving multiplication.	Count in steps of different sizes.	5 x 4 = 15 is the same as 5 + 5 + 5+ 5 = 20 		The main concept to get across is that when you multiply you are repeatedly adding the same number again and again. Calculate the answer using concrete objects, pictorial representations and arrays with the support of an adult.
STAGE 2 -Show that multiplication of two numbers can be done in any order. -Calculate statements for multiplication using the x sign.	Count in steps of… Knowledge of times tables. Add two or more single-digit numbers mentally.	Array 	Number line 	Read out the calculations as: 3x4 '3, multiplied 4 times' Understand that this is a group of 3, repeated 4 times. Use an array to model the concept. Emphasise that children don't count individual dots, but count up in the appropriate steps. This can lead onto children representing their counting on a number line. Use Numicon tiles to turn model around, demonstrating that multiplication can be done in any order.
STAGE 3 Write and calculate mathematical statements for multiplication including for two-digit numbers times one-digit numbers.	recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables Doubling & Halving rapid recall Partition number into tens and units Children should be able to multiply by ten mentally.	Can introduce informally using partitioning '13 x 6 is the same as 10 x 6 and 3 x 6 (60 + 18) = 78	Use repeated Numicon arrays 	It is important that doubling and halving are taught independently to other mental strategies for multiplying and dividing. It is not appropriate for children to use arrays or number lines to multiply by 2. Children should be able to double, even large numbers through partitioning, mentally. Through doubling, they connect the 2, 4 and 8 multiplication tables.
		Work towards standard method 2 4 x 3 = (3 x 4) + (3 x 20) = = 12 + 60 = 72		Use MOVING DIGITS ITP / place value chart: Emphasise the DIGITS MOVE, not adding on a zero when x by 10 (otherwise, when working with money, children will put £1.75 x 10 = £1.750!!!)

<table>
<tr>
<td><u>STAGE 4</u>
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.</td>
<td>Recall multiplication and division facts for multiplication tables up to 12 × 12
Partition number into multiples of one hundred, ten and one.</td>
<td colspan="3">… Then Grid Method
<table>
<tr><td>x</td><td>10</td><td>3</td></tr>
<tr><td>6</td><td>60</td><td>18</td></tr>
</table>
60 + 18 = 78

Then 242 x 7 = 1694
200 40 2
<table>
<tr><td>X 7</td><td>1400</td><td>280</td><td>14</td></tr>
</table>
1400
280
14
1694</td>
<td>The grid method allows children to use known number facts to solve multiplication problems.

Pupils need to work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
Pupils add combinations of whole numbers using the column method.</td>
</tr>
<tr>
<td><u>STAGE 5</u>
Multiply numbers up to 4 digits by a one- or two-digit number.</td>
<td></td>
<td colspan="2">24 x 14 = 336
<table>
<tr><td>x</td><td>20</td><td>4</td><td></td></tr>
<tr><td>10</td><td>200</td><td>40</td><td>=240</td></tr>
<tr><td>4</td><td>80</td><td>16</td><td>= 96</td></tr>
</table>
240
+96
336</td>
<td>Compact (standard method)
2 4
X 3
7 2
1
(Use familiar calculations to show links)</td>
<td>Children can begin to use the compact methods when they are comfortable with solving the multiplications mentally.
-multiply 3 digit and 4 digit numbers by a number (between 11 and 20) by the end of Year 5).</td>
</tr>
<tr>
<td><u>STAGE 5 / 6</u>
Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.</td>
<td>Knowledge of times tables facts up to 12x12</td>
<td colspan="2">14.53 x 4 = 58.12
<table>
<tr><td>X</td><td>10</td><td>4</td><td>0.5</td><td>0.03</td></tr>
<tr><td>4</td><td>40</td><td>16</td><td>2</td><td>0.12</td></tr>
</table>
40
16
2
0.12
58.12</td>
<td>Long Multiplication

124 × 26 becomes
<table>
<tr><td></td><td>1</td><td>2</td><td></td></tr>
<tr><td></td><td>1</td><td>2</td><td>4</td></tr>
<tr><td>×</td><td></td><td>2</td><td>6</td></tr>
<tr><td></td><td>7</td><td>4</td><td>4</td></tr>
<tr><td>2</td><td>4</td><td>8</td><td>0</td></tr>
<tr><td>3</td><td>2</td><td>2</td><td>4</td></tr>
<tr><td>1</td><td>1</td><td></td><td></td></tr>
</table>
Answer: 3224</td>
<td>Pupils use a formal written method, including long multiplication for two-digit numbers. Continue to use 0 as place holder if necessary. Show column format alongside grid method to show it is the same but with a different layout.

Pupils use short multiplication to multiply a one digit number by a number with one or two decimal places including money.</td>
</tr>
</table>